In the following report, Hanover Research examines the effectiveness of integrated mathematics curricula at the secondary level. The report also describes key considerations in transitioning from a traditional curriculum to an integrated curriculum, including communication with stakeholders and professional development.
# TABLE OF CONTENTS

**Executive Summary and Key Findings** ................................................................. 3
  - Introduction .......................................................................................................... 3
  - Key Findings ........................................................................................................ 3
**Section I: Integrated Mathematics Effectiveness** ............................................. 5
  - Purpose of Integrated Math .................................................................................. 5
  - Studies Examining Integrated Math ..................................................................... 7
**Section II: Best Practices in Implementation** ................................................... 12
  - Communication .................................................................................................... 12
  - Professional Development .................................................................................... 15
EXECUTIVE SUMMARY AND KEY FINDINGS

INTRODUCTION
Integrated math curricula incorporate content from across mathematical disciplines into a single course. Secondary-level integrated math combines content from algebra, geometry, and statistics into a three-course sequence. This sequence of Math I, Math II, and Math III replaces the traditional course sequence of Algebra I, Geometry, and Algebra II.

Although integrated math is an established practice, a 2014 article from Education Week describes the Common Core State Standards (CCSS) as the “catalyst” for a growing integrated math movement in the United States. North Carolina, West Virginia, and Utah mandate all public high schools teach integrated math, and most other states allow districts to choose between integrated and traditional course sequences.

In the following report, Hanover Research evaluates the evidence supporting integrated math and describes best practices for communicating curricular changes and providing professional development to guide implementation. The report is organized into the following sections:

- **Section I** reviews the existing literature on integrated secondary mathematics, presenting results from six studies that examine the effects of integrated math.
- **Section II** presents best practices for the implementation of an integrated secondary mathematics curriculum, focusing on communication to district stakeholders and professional development for school staff.

KEY FINDINGS

- **Studies show that integrated math instruction positively impacts mathematics achievement.** Four experimental studies examined in this report found that students enrolled in integrated math courses outperformed students enrolled in traditional math courses. Although the countries that outperform the United States in mathematics typically rely on an integrated curriculum, empirical evidence has not definitively attributed this variation in achievement levels to the choice of course sequence.

- **Integrated math courses help students develop a deeper understanding of mathematical concepts and an appreciation for the relevance of these concepts to the real world.** Often, real world math problems incorporate concepts from multiple subjects; likewise, integrated math courses teach students to think about math in an applied, interconnected manner. Students in integrated courses gain a greater understanding of mathematical concepts due to the repetition of concepts.

---

2. Ibid.
throughout multiple courses and in the context of related topics (e.g., algebra, geometry, and statistics).

- **Schools benefit from offering forums to communicate with parents and gather parent feedback on the transition to an integrated mathematics course sequence.** Through parent workshops, educators can expose parents to content from the new curriculum to demonstrate how the lessons extend beyond what is taught in traditional math courses. Through parent surveys, the district or school may also learn about parents’ concerns and questions regarding the transition. By including parents in the transition process, administrators may garner greater support for the change.
SECTION I: INTEGRATED MATHEMATICS EFFECTIVENESS

This section reviews the existing literature on integrated secondary mathematics, incorporating information from scientific studies as well as education publications.

PURPOSE OF INTEGRATED MATH

Since the California State Board of Education adopted the Common Core State Standards (CCSS) in 2012, educators have worked to adapt to a changing instructional landscape. These rigorous standards require curriculum changes to both traditional math courses and integrated math courses:

The standards call for learning mathematical content in the context of real-world situations, using mathematics to solve problems, and developing “habits of mind” that foster mastery of mathematics content as well as mathematical understanding. The standards for Kindergarten through Grade 8 prepare students for higher mathematics. The standards for higher mathematics reflect the knowledge and skills that are necessary to prepare students for college and careers and productive citizenship.

The California State Board of Education allows schools to choose between traditional and integrated pathways and presents the standards in both formats. The curricular paths model those in Appendix A of the Common Core State Standards. Although either approach to mathematics instruction is acceptable, the California State Board of Education suggests, “the integrated pathway presents higher mathematics as a connected subject.”

Integrating the core mathematics subjects of algebra and geometry offers several benefits. First, students recognize the interconnected nature of mathematical topics and see their application in context. This recognition deepens their understanding of each subject. Second, students gain repeated exposure to key topics in subsequent courses, rather than experiencing each topic in isolation. This consistency allows students to “systematically build proficiency in each domain.” Lastly, integrated math courses typically emphasize

---

4 Ibid., p. 58.
problem solving as a method of instruction, which relates mathematical topics to real-world situations.\textsuperscript{9}

Advocates of an integrated math curriculum note that the vast majority of countries other than the United States employ integrated approaches to mathematics instruction, and many of these countries outperform the U.S. on international achievement tests.\textsuperscript{10} The United States ranks number 36 in the world in math according to the Organization for Economic Cooperation and Development (OECD) Program for International Student Assessment (PISA).\textsuperscript{11} Nearly all of the countries ranked more highly than the U.S. follow an integrated math curriculum.\textsuperscript{12} The international curricula designs integrate algebra, geometry, probability, statistics, and discrete mathematics to provide a “broad and integrated” program rather than the “narrow and compartmentalized structure of traditional programs.”\textsuperscript{13}

In December 2013, the non-profit Consortium for Mathematics and Its Applications sponsored a discussion of the CCSS with a group of 11 senior mathematicians, teachers, statisticians, teacher educators, and curriculum developers with “extensive experience in school mathematics innovation.” The group was asked to offer a set of progressive recommendations, one of which included integrated mathematics instruction as a method to advance U.S. achievement in math.\textsuperscript{14} As the president of the National Council of Teachers of Mathematics has similarly argued, the U.S. cannot accurately compare its performance to that of other countries with such a different curriculum.\textsuperscript{15}

Although some have concluded that students in foreign countries score better on math assessments than students in the U.S. largely due to the integrated curriculum, little empirical evidence supports this assertion. In 2008, the U.S. Department of Education’s National Mathematics Advisory Panel reported a lack of success in identifying such evidence:

A search of the literature did not produce studies that clearly examined whether an integrated approach or a single-subject sequence is more effective for algebra and


\textsuperscript{14} Ibid.

more advanced mathematics course work. The Panel finds no basis in research for
preferring one or the other.\textsuperscript{16}

**STUDIES EXAMINING INTEGRATED MATH**

Given the controversial nature of an integrated curriculum, several studies have examined
the effects of integrated math. Hanover reviews six of these studies, as demonstrated in
Figure 1.1. It should be noted that randomized control trials are not typically possible in
educational settings because neither students nor teachers are randomly assigned to classes
or interventions. A discussion of each study follows the figure.

**Figure 1.1: Studies Evaluating Integrated Math**

<table>
<thead>
<tr>
<th>Authors</th>
<th>Design</th>
<th>Population</th>
<th>Length</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finkelstein, et al.</td>
<td>Data analysis</td>
<td>24,279</td>
<td>6 years</td>
<td>Students who take Algebra I before they are fully prepared may never reach proficiency in the subject.</td>
</tr>
<tr>
<td>Schoen and Hirsch</td>
<td>Quasi-experimental</td>
<td>1,050</td>
<td>2 years</td>
<td>Students using Core-Plus Mathematics, an integrated curriculum, scored significantly higher on multiple achievement metrics than students using a traditional curriculum.</td>
</tr>
<tr>
<td>Tauer</td>
<td>Randomized control trial</td>
<td>120</td>
<td>2 years</td>
<td>Students enrolled in an integrated math program were more likely to achieve proficiency on the Grade 10 Kansas State Mathematics Assessment and enroll in senior mathematics classes than their peers in traditional math classes.</td>
</tr>
<tr>
<td>Grouws et al.</td>
<td>Quasi-experimental</td>
<td>2,161</td>
<td>1 year</td>
<td>Students who studied from Core-Plus I, an integrated course, scored significantly higher on three achievement tests than students taking an Algebra I course.</td>
</tr>
<tr>
<td>Tarr, et al.</td>
<td>Quasi-experimental</td>
<td>3,258</td>
<td>3 years</td>
<td>Students who studied from Core-Plus II, an integrated course, scored significantly higher on standardized achievement tests than students taking a Geometry course.</td>
</tr>
<tr>
<td>Tarr, et al.</td>
<td>Data analysis</td>
<td>2,621</td>
<td>3 years</td>
<td>Curriculum type (i.e., subject-based or integrated) is significantly correlated with student achievement.</td>
</tr>
</tbody>
</table>

**Finkelstein, et al. (2012)**

This 2012 study of over 24,000 students in California demonstrates the need for reform
given the inadequacies of the current secondary math sequence. In particular, the study showed that only 34 percent of Grade 11 students in the state tested proficient in Algebra I,

although over half of the students took Algebra I for the first time in Grade 8 and approximately 20 percent took Algebra I for the first time in Grade 9.\textsuperscript{17} Students who repeated Algebra I in subsequent years showed "little to no improvement."\textsuperscript{18}

The analysis presented in this report clearly shows that some students, those with Grade 7 California Standards Test (CST) scores at the level of Proficient or higher, continue to excel in math throughout high school, regardless of when they take Algebra I. On the other hand, it also shows that students who move too quickly through their math sequence in middle school (i.e., taking Algebra I before they are fully prepared) \textit{never} reach the level of Proficient on the Algebra I CST, an outcome that has direct consequences for their performance in higher-level math courses, and, ultimately, for their placement in postsecondary math courses should they go on to higher education.\textsuperscript{19}

The report suggested that student readiness for Algebra I is more important than the grade in which students take the course. Students who repeat Algebra I rarely make progress in attaining proficiency, and, therefore, the study authors recommended that schools encourage students to take more time to develop the foundational concepts and skills prior to taking Algebra I rather than taking Algebra I earlier with the potential to repeat it in later grades.\textsuperscript{20}

\textbf{Schoen and Hirsch (2002)}

One of the most widely used curricula for integrated math is Core-Plus Mathematics. Core-Plus Mathematics is a four-year curriculum that follows an integrated math sequence of "interwoven strands of algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics."\textsuperscript{21} Instruction centers around "collaborative small-group investigations of problem situations, followed by teacher-led whole-class summarizing activities."\textsuperscript{22} A 2002 study of the Core-Plus curriculum conducted across 11 high schools examined achievement outcomes for 1,050 students who were in either a Core-Plus Mathematics classroom or a traditional classroom.\textsuperscript{23} The study meets the U.S. Department of Education’s What Works Clearinghouse (WWC) evidence standards with reservations.\textsuperscript{24} For students using the Core-Plus curriculum, the study reported positive


\textsuperscript{19} Finkelstein et al., Op. cit., p. 42.

\textsuperscript{20} Ibid., p. 30.


\textsuperscript{22} Ibid.


\textsuperscript{24} According to the What Works Clearinghouse, “Meets standards with reservations is the middle possible rating for a group design rating reviewed by the WWC. Studies receiving this rating provide a lower degree of confidence that an observed effect was caused by the intervention. Strong quasi-experimental designs may receive this rating.” http://ies.ed.gov/ncee/wwc/Glossary.aspx
effects on SAT math scores, the Iowa Tests of Educational Development for Grade 9 mathematics, and subtests for Core-Plus Mathematics. The WWC calculated an average improvement of 15 percentile points for students in the intervention group over the students in the control group.25

**Tauer (2002)**
The findings of this scientifically rigorous study corroborate those of Schoen and Hirsch. A 2002 comparative study of 120 students in Derby School District (DSD) in Derby, Kansas compared the performance of students randomly assigned to either a traditional math curriculum or the Core-Plus curriculum. The study authors randomly assigned half of the study volunteers to the control group and half to the treatment group, although most of the participants would have been recommended to take Algebra I in Grade 9. The two groups had almost identical average scores on the Grade 7 Kansas State Mathematics Assessment.

In Grades 9 and 10, the control group enrolled in Algebra I and Geometry using the district’s existing curriculum, and the treatment group completed Math I using the Core-Plus Mathematics curriculum. Teachers using the Core-Plus curriculum received training on the new instructional materials.

Students enrolled in the integrated math program were more likely to score “proficient” or higher on the Grade 10 Kansas State Mathematics Assessment than were their peers in traditional math classes (58.2 percent compared to 46.5 percent). In addition, 79.1 percent of students in the Core-Plus Mathematics group enrolled in a “fourth-year elective college preparatory mathematics class,” compared to 46.5 percent of students in the traditional group.26

**COSMIC Project**
The following three studies originated from the *Comparing Options in Secondary Mathematics: Investigating Curriculum (COSMIC)* research project, funded by the National Science Foundation. The project evaluated secondary school mathematics by comparing students enrolled in a traditional math curriculum and an integrated math curriculum.27

**Grouws, et al. (2013)**
The first phase of the COSMIC project examined student achievement in the first year of a secondary math sequence. Published in 2013, this study of 2,161 students in 10 schools across five states examined the effects of both subject-specific (e.g., Algebra I) and integrated (e.g., Math I) math curricula. Students who studied from the integrated curriculum scored significantly higher than students who studied from the subject-specific curriculum on three assessments: a test of common objectives, a problem-solving and

reasoning test, and a standardized achievement test. The test of common objectives assessed learning targets common to both curriculum types, while the problem solving and reasoning test assessed mathematical reasoning on grade-level appropriate topics. The study relied on the Iowa Test of Educational Development as a standardized achievement test. The COSMIC project employed these three assessments in subsequent studies.

**Tarr, et al. (2013)**
The second phase of the COSMIC project examined study achievement after the second year of the secondary math sequence. Published in 2013, this study of more than 3,000 high school students nationwide found that students enrolled in the integrated mathematics course “scored significantly higher on standardized tests administered to all participating students, after controlling for many teacher and student attributes.” Furthermore, the study authors concluded that students with a history of high academic achievement “benefitted more from the integrated mathematics program than students who studied from the traditional curriculum.”

The study examined end-of-year outcomes for students in 11 high schools across five states enrolled in either Math II, an integrated course, or Geometry, a traditional subject-specific course. The students independently chose which course to complete and were not tracked for one course or the other based on prior achievement. The study standardized students’ scores on state-mandated Grade 8 tests the prior year to calculate growth.

The students were mostly in their second year of high school, with approximately 20 percent of each group in Grade 9 and less than 10 percent of each group in Grades 11 and 12. African American, Hispanic, and IEP students were more likely to enroll in Geometry than in Math II.

**Tarr, et al. (2010)**
The authors conducted a related study as a precursor to those described above. Published in 2010, this longitudinal, five-state study of 2,621 secondary school students in integrated and subject-specific math courses analyzed National Assessment of Educational Progress (NAEP) scores controlling for prior achievement. Curriculum type (e.g., integrated or

---


30 Ibid.


32 Ibid., p. 699.
subject-specific) was significantly correlated with student scores on project-development assessments as well as the Iowa Test of Educational Development for mathematics when controlling for percentage free and reduced-price lunch eligibility, with students in integrated curricula showing greater gains than students enrolled in traditional math courses.  

The students in subject-specific curricula used textbooks from Holt, Prentice Hall, Glencoe, McDougal Littell, and HRW in Algebra I, Geometry, and Algebra II. The students in the integrated curricula used the Core-Plus curriculum for Math I, Math II, and Math III. Subject-specific lessons typically included teacher-led, whole-class discussions, whereas integrated subject lessons typically incorporated small-group work.

---

34 Ibid., p. 5.
SECTION II: BEST PRACTICES IN IMPLEMENTATION

This section reviews best practices in the implementation of an integrated secondary-level mathematics curriculum, focusing on communication to district stakeholders and professional development for school staff.

COMMUNICATION

According to the Interactive Mathematics Program, an integrated math curriculum developed in collaboration with the National Science Foundation, parent communication is integral to successful curriculum adoption:

The decision to implement [an integrated math] curriculum should involve parents and students. Parent-teacher organization meetings and back-to-school nights provide excellent forums for starting the conversation about the need for change. This discussion can build on the fact that the world is changing and that the mathematical expectations facing adults entering the twenty-first century will be different from what was required of their parents. It is important to stress that the new programs do not shortchange fundamentals, but rather go beyond rote learning to encourage a deep understanding of the meaning and uses of mathematics. Having parents reflect on their own, often negative, experiences with school mathematics can help them understand why change is required. An interactive [sample lesson] can give them a vision of how positive and exciting a mathematics program can be.35

The following descriptions elaborate on communication initiatives in three school districts following the decision to implement an integrated math curriculum.

Sacramento City Unified School District

Sacramento City USD approved integrated math courses in March 2015 to start in the 2015-2016 school year.36 The district solicited feedback from teachers, school leaders, parents, students, and community members in selecting which instructional materials to use for the integrated pathway; using that information, an instructional materials evaluation committee decided to adopt Walch’s Common Core mathematics.37 According to the publisher, the materials “were created with direct input from California teachers and curriculum leaders.”38 Figure 2.1 describes the committee’s evaluation of the curriculum.

---

The district plans to offer parent workshops “on effective use of the adopted instructional materials.”

**San Diego Unified School District**

In 2014, San Diego County gave local school districts the option of keeping a traditional high school math course pathway (i.e., Algebra, Geometry, and Algebra II) while adapting the curriculum to meet the Common Core State Standards, or implementing an integrated math sequence. Ultimately, most districts in the county chose to transition to integrated math courses. San Diego USD transitioned to the CCSS and an integrated math curriculum in the 2014-2015 school year. The decision process lasted two years, starting with department chair discussions and including an examination of traditional and integrated math pathways and a review of California state guidelines and expectations. The district experienced substantial parent pushback and hosted parent meetings to “ease concerns.” The district posts answers to frequently asked questions on its website (see Figure 2.2 for an example), which includes an explanation of how administrators decided to implement the integrated curriculum. According to the site, “it was a collective decision of teachers, administrators,

---

### Figure 2.1: Strengths and Challenges of Integrated Math Curriculum

<table>
<thead>
<tr>
<th>Strengths of Walch’s Curriculum:</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Coherence</td>
</tr>
<tr>
<td>✓ Modeling</td>
</tr>
<tr>
<td>✓ Design</td>
</tr>
<tr>
<td>✓ Standards for mathematical practice</td>
</tr>
<tr>
<td>✓ Problems/Practice</td>
</tr>
<tr>
<td>✓ Concept building</td>
</tr>
<tr>
<td>✓ Discussion guidelines</td>
</tr>
<tr>
<td>✓ Different options</td>
</tr>
<tr>
<td>✓ Online teacher portal for learning management and resources</td>
</tr>
<tr>
<td>✓ Publisher Flexibility</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Challenges of Walch’s Curriculum:</th>
</tr>
</thead>
<tbody>
<tr>
<td>✗ Aesthetics</td>
</tr>
<tr>
<td>✗ Relevance to students</td>
</tr>
<tr>
<td>✗ Lots of text</td>
</tr>
<tr>
<td>✗ Manipulatives</td>
</tr>
<tr>
<td>✗ Student digital resources limited to downloadable and CD formats</td>
</tr>
<tr>
<td>✗ Online assessments are primarily multiple choice</td>
</tr>
</tbody>
</table>

Source: Sacramento City Unified School District


40 Ibid., p. 5.


42 Ibid.


44 Ibid.

the Math Task Force, and the University of California-San Diego that integrated curriculum would best meet the needs of our students.\(^{46}\)

**Figure 2.2: San Diego USD Parent FAQ**

<table>
<thead>
<tr>
<th>Why did the Grades 6-12 math course and sequence change?</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students should have learning experiences that make sense as they move from course to course to ensure that they will be college-ready by the end of high school. The new course sequence in mathematics creates a coherent program, focusing deeply on fewer concepts and concentrating on bigger ideas. Students gain a stronger foundation from which to build their mathematical competency. At SDUSD, all secondary schools offer this new course sequence (that is, the order in which students are taking their math classes) so that students can build upon their conceptual understanding from year to year. Rather than taking a different set of standards each year, students will experience a progression of coherent standards, each building upon those of the previous year.</td>
</tr>
</tbody>
</table>

Source: San Diego Unified School District\(^{47}\)

**Metropolitan Nashville Public Schools**

Metropolitan Nashville Public Schools (MNPS) announced in January 2015 that the district would adopt an integrated math curriculum starting in the 2015-2016 school year.\(^{48}\) To explain the decision, MNPS states, “integrated math will give students an edge by teaching in the same way as countries that are excelling in math.”\(^{49}\) MNPS maintains a website that explains the new integrated math pathway. The website lists an email address and a phone number for comments and questions on integrated math and also offers parent one-pagers on the transition in four languages. The school system hosted six parent meetings on integrated math to provide more information on the new curriculum.\(^{50}\)

---


\(^{47}\) Ibid.


PROFESSIONAL DEVELOPMENT

As noted in a report by Dean Fixsen of the National Implementation Research Network, evidence-based practices cannot impact student achievement without effective implementation processes.\textsuperscript{51} A key factor in successful implementation is sufficient training for those groups enacting reforms. Research suggests that highly effective professional development focuses on action and interaction over an extended time span and relies on concrete training activities specific to the teacher’s role.\textsuperscript{52}

San Diego USD provides summer institutes and additional professional development to prepare elementary and secondary teachers for the change in the math curriculum. The district also offers professional development specifically for the new instructional materials, as well as “scope and sequences, pacing guides, assessments, and other instructional supports for all of these courses.”\textsuperscript{53}

The Sacramento City USD plans to offer training for teachers and other leaders “to effectively utilize the adopted instructional materials including addressing the challenge areas identified by the instructional materials committee.”\textsuperscript{54} As the districts are currently implementing integrated math, these initiatives are in progress.

Experts also highlight the value of offering tailored training to various school staff members. The Mathematics Program Resource Center emphasizes the importance of educating school counselors on new curricula, given their direct interaction with students and parents during the transition:

School counselors play a major role by informing students and parents about curricular changes, so it is essential that counselors at the high school and its feeder schools be knowledgeable [about the curriculum changes] and be able to communicate that [integrated math] is a rigorous college preparatory mathematics program that is appropriate for college-bound students.\textsuperscript{55}

\textsuperscript{54} “Board of Education Executive Summary: Adoption of Mathematics Instructional Materials,” Op. cit., p. 5.
PROJECT EVALUATION FORM

Hanover Research is committed to providing a work product that meets or exceeds partner expectations. In keeping with that goal, we would like to hear your opinions regarding our reports. Feedback is critically important and serves as the strongest mechanism by which we tailor our research to your organization. When you have had a chance to evaluate this report, please take a moment to fill out the following questionnaire.


CAVEAT

The publisher and authors have used their best efforts in preparing this brief. The publisher and authors make no representations or warranties with respect to the accuracy or completeness of the contents of this brief and specifically disclaim any implied warranties of fitness for a particular purpose. There are no warranties that extend beyond the descriptions contained in this paragraph. No warranty may be created or extended by representatives of Hanover Research or its marketing materials. The accuracy and completeness of the information provided herein and the opinions stated herein are not guaranteed or warranted to produce any particular results, and the advice and strategies contained herein may not be suitable for every partner. Neither the publisher nor the authors shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages. Moreover, Hanover Research is not engaged in rendering legal, accounting, or other professional services. Partners requiring such services are advised to consult an appropriate professional.